

本考科可使用掌上型計算機

1. Find the total derivatives of the following expressions.

(a) (8 points) $y = (\sin x)^x$

(b) (8 points) $y = \frac{\ln u}{v}$

2. Solve the following first-order linear differential equations.

(a) (10 points) $\frac{dy}{dx} + 3x^2y = 0$

(b) (10 points) $\frac{dy}{dx} + \frac{1}{x}y = \sqrt{x}$

3. Find the fourth Taylor polynomial for

(a) (8 points) e^{x^2}

(b) (8 points) $\log x$ in powers of $x - 2$

4. Find

(a) (8 points) $\lim_{x \rightarrow \infty} x \log \left(1 + \frac{1}{x} \right)$

(b) (8 points) $\lim_{n \rightarrow \infty} \frac{n^{10} + \sqrt{n} \log n}{n! + n^2 \log n}$

(c) (8 points) $\lim_{n \rightarrow \infty} n^{\frac{1}{n}}$

5. Evaluate the following integrals.

(a) (8 points) $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx$

(b) (8 points) $\int x\sqrt{2x+3} dx$

(c) (8 points) $\int e^x \cos x dx$