

本考科禁用計算機

1. Find necessary and sufficient conditions for the numbers A and B to make the LP problem

$$\begin{aligned} \max \quad & x_1 + x_2 \\ \text{s.t.} \quad & Ax_1 + Bx_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{aligned}$$

- (a) (5 points) have an optimal solution,
(b) (5 points) be infeasible,
(c) (5 points) be unbounded.

2. Consider the LP

$$\begin{aligned} \min z = \quad & 5x_1 + 3x_2 - x_3 \\ \text{s.t.} \quad & x_1 - 2x_2 + x_3 \geq 2 \\ & -2x_1 - x_2 - 3x_3 \geq -10 \\ & x_1 + x_2 + x_3 = 5 \\ & x_j \geq 0, j = 1, \dots, 3 \end{aligned}$$

- (a) (7 points) Write the dual problem.
(b) (10 points) Solve the primal problem using the two-phase method, and identify the optimal primal and dual solutions.
(c) (10 points) Solve the dual problem directly using the two-phase method, and identify the optimal primal and dual solutions.

3. (20 points) For this distribution problem, supply and demand data, direct shipping costs from suppliers to customers, and transshipment costs to and from a terminal are given in the matrix.

		Customer			Supply	
		5	6	7		
Supplier	1	5	4	3	2	15
	2	1	2	3	2	15
	3	5	5	4	3	15
	4	6	1	4	3	15
Terminal		3	1	2		
Demand		17	15	20		

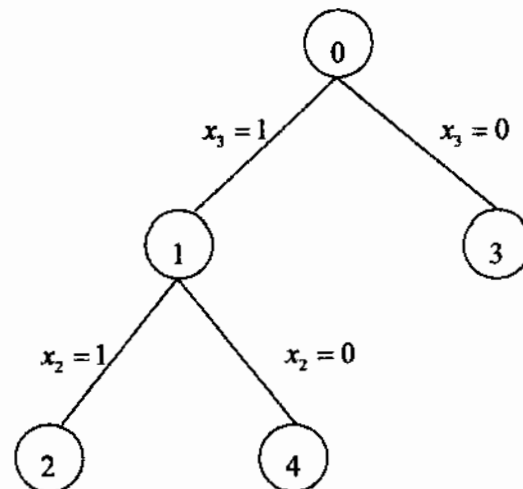
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In addition, the following restrictions must be met.

- All demands must be satisfied.
- Suppliers 1 and 2 together can ship no more than a total of 10 units to customer 5 directly.
- No shipping link can handle more than seven units.
- The transshipment terminal can handle no more than 15 units.

Set up and solve the network flow model for this problem.

4. The following is the complete branch-and-bound tree for an ILP over decision variables $x_1, \dots, x_4 = 0$ or 1 .



- (a) (6 points) List the partial solutions associated with each node of the tree.
 (b) (6 points) Which nodes were branched and which terminated?
 (c) (6 points) Identify the nodes of the tree that have $x = (0, 1, 0, 1)$ as a feasible completion.

5. Solve the following problems by DP.

- (a) (10 points)
$$\max z = 8x_1 + 7x_2$$

 s.t.

$$2x_1 + x_2 \leq 8$$

$$5x_1 + 2x_2 \leq 15$$

$$x_1, x_2 \geq 0 \text{ and integer}$$

- (b) (10 points)
$$\max z = 7x_1^2 + 6x_1 + 5x_2^2$$

 s.t.

$$x_1 + 2x_2 \leq 10$$

$$x_1 - 3x_2 \leq 9$$

$$x_1, x_2 \geq 0$$