招生學年度	九十七	招生類別	碩士班、碩士學位學程
系所班別	全球運籌管理研究所		
科目	作業研究		
意事項	可使用掌上型計算機		

1. Write the dual problems of the following two LP problems.

(a) (10 points)

$$\max \sum_{r=1}^{s} u_r y_{rk}$$

$$\sum_{r=1}^{s} u_r y_{rj} \le \sum_{i=1}^{m} v_i x_{ij} \quad (j=1,2,\ldots,n)$$

$$\sum_{i=1}^{m} v_i x_{ik} = 1$$

$$v_i \ge 0 \quad (i=1,2,\ldots,m)$$

$$u_r \ge 0 \quad (r=1,2,\ldots,s)$$

Note that k is one specific element of $\{1,2,\ldots,n\}$.

(b) (15 points)

$$\max \sum_{r=1}^{s} u_r y_{rk} + \sigma$$

$$\sum_{r=1}^{s} u_r y_{rj} + \sigma \le \sum_{i=1}^{m} v_i x_{ij} \quad (j=1,2,\ldots,n)$$

$$\sum_{i=1}^{m} v_i x_{ik} = 1$$

$$v_i \ge 0 \quad (i=1,2,\ldots,m)$$

$$u_r \ge 0 \quad (r=1,2,\ldots,s)$$

σ unrestricted

Note that k is one specific element of $\{1,2,\ldots,n\}$.

2. (25 points) Suppose we are given a set J of candidate facility locations and a set I of demand points. Let h_i be the demand at node i, P be the number of facilities to locate, and N_i be the set of all candidate locations that can cover demand node i. Formulate an IP to select P facility locations to maximize the total covered demands.

3. Consider the LP

$$\max z = \sum_{j=1}^{n} c_j x_j$$

$$s.t.$$

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \quad (i=1,2,\ldots,m)$$

$$x_j \ge 0 \quad (j=1,2,\ldots,n)$$

(a) (5 points) Write the dual problem.

(b) (5 points) State the weak duality property.

招生學年度	九十七	招生類別	碩士班、碩士學位學程
系所班別	全球運籌管理研究所		
科目	作業研究		
注意事項	可使用掌上型計算機		

(c) (5 points) State the strong duality property.
(d) (5 points) State the optimality property.
(e) (5 points) State the unboundedness property.

4. Consider the nonlinear program

$$\max z = x_1 \times x_2$$
$$s.t.$$
$$x_1 + x_2 = 8$$

(a) (10 points) Solve the problem using the Karush-Kuhn-Tucker (KKT) conditions.
(b) (15 points) Verify that the solution is a saddle point.