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國立東華大學招生考試試題

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招生學年度	九十八	招生類別	碩士班
系所班別	全球運籌管理研究所		
科目	微積分		
注意事項	本考科可使用掌上型計算機		

Notes. The whole paper is of 100 points. For every question, show your reasoning and steps to avoid losing a significant number of points.

#1. It is known that  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ .

(a). (5 points) Find  $\lim_{x \rightarrow \infty} \sqrt{x} \sin \frac{1}{\sqrt{x}}$ .

(b). (5 points) Find  $\lim_{x \rightarrow 0} \frac{5x + \sin 3x}{x - 6 \sin x}$ .

(c). (5 points) Find  $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$ .

(d). (5 points) Find  $\lim_{x \rightarrow 0} \left( x - \frac{1}{\sqrt{x}} \right) \sin(\sqrt{x})$ .

#2. In the following,  $m$  is a fixed positive constant and  $n$  takes up integer values as it tends to infinity.

(a). (5 points) Find  $\lim_{n \rightarrow \infty} \sqrt{n^2 + 3n} - n$ .

(b). (10 points) Find  $\lim_{n \rightarrow \infty} \frac{m^n n!}{n^n}$ . (Note. From Stirling's formula,  $n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$ .)

#3. Let  $\{a_n\}$  be a real-valued sequence such that  $a_1 = \sqrt{3}$  and  $a_{n+1} = \sqrt{3 + a_n}$  for  $n \geq 1$ .

(a). (5 points) Show that  $a_n$  is increasing in  $n$ .

(b). (5 points) Show that  $a_n$  is bounded above for all  $n$ .

(c). (5 points) From (a) and (b),  $\lim_{n \rightarrow \infty} a_n$  exists. Find its value.

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#4. Find  $\frac{dy}{dx}$  from the following functions.

(a). (5 points)  $e^{\sin x} + y \ln x = x^2 y.$

(b). (5 points)  $y = \tan x^x.$  (Note.  $\frac{d \tan x}{dx} = \sec^2 x.$ )

#5(a). (5 points) Write down the Taylor expansion of  $\cos(x + \delta)$  around  $x$  for small  $\delta$ .

(b). (5 points) Estimate  $\cos\left(\frac{\pi}{3} + 0.01\right)$  by the first three terms of its Taylor expansion,

given that  $\cos \frac{\pi}{3} = \frac{1}{2}$  and  $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}.$

#6. Find the indefinite integral of the following functions where you can ignore the constant term.

(a). (5 points)  $\int e^x \sin x \, dx.$

(b). (5 points)  $\int x/(x^2 - 3)^2 \, dx.$

(c). (5 points)  $\int 1/[(x-3)(x+2)] \, dx.$

#7. Let  $f(x) = x$ ,  $g(x) = x^2$ ,  $S$  be the region between  $f(x)$  and  $g(x)$  in the first quadrant.

(a). (5 points) Find the area of  $S$ .

(b). (5 points) Find the circumference of  $S$ . You can show your procedure without evaluating the exact value.

(c). (5 points) Find the volume of the object created by rotating  $S$  around the  $x$  axis.